The selection of mechanical actuators based on performance indices

By J. E. Huber, N. A. Fleck and M. F. Ashby

Department of Engineering, Cambridge University, Trumpington Street, Cambridge CB2 1PZ, UK

A method is presented for selecting the type of actuator best suited to a given task, in the early stages of engineering design. The selection is based on matching performance characteristics of the actuator, such as force and displacement, to the requirements of the given task. The performance characteristics are estimated from manufacturers' data and from simple models of performance limitation such as heat generation and resonance. Characteristics are presented in a graphical form which allows for a direct and systematic comparison of widely different systems of actuation. The actuators considered include man-made actuators (such as hydraulic, solenoid and shape memory alloy) and naturally occurring actuators (such as the muscles of animals and plants).

1. Introduction

Actuators provide the driving force and motion for a variety of natural and man-made requirements; typical examples are listed in table 1. In each case a mechanical action is activated in response to a control signal. Naturally occurring actuators include the muscles of animals and plants, and man-made actuators include hydraulics, pneumatics and solenoids. Other man-made actuators, such as piezoelectric, shape memory alloy and magnetostrictive devices, are based on shape-changing materials; these are used increasingly in novel applications. For example, piezoelectric actuators are used in precision positioning devices such as the reading heads in video cassette recorders and compact disc players. They have been proposed for active materials and structures (Shen 1994), adjustable aerodynamic surfaces (Barrett 1992), vibration damping (Crawley & de Luis 1987) and noise cancellation. Shape memory alloys have found applications mainly where a single contracting stroke is required, such as in pipe couplings and orthodontic wires; cyclic applications include actuators in robot end effectors (Venison 1986; Furnya & Shimada 1991) and satellite structure deployment (McDonald Schetky 1991). Magnetostrictive actuators have found relatively few applications; suggested uses include vibration isolation and active aerodynamic surfaces (Bothwell *et al.* 1995). A description of each of the classes of actuator discussed in this article is given in the appendix. Energy storage devices such as springs, flywheels and weights are not included in the analysis.

   Given the wide variety of existing applications and actuators, some means of matching the requirements of an application to the performance characteristics of an actuator is desirable. The mechanical requirements of an application can be expressed

Table 1. *Typical applications of actuators*

aerospace	automotive	industrial equipment
flight control surfaces landing gear movement	braking tappets	automation equipment numerically controlled machines
nose wheel steering air brakes powered doors/hatches	active suspension active engine mounts airbag deployment	presses lifting equipment
electrical goods	developing technologies	instrumentation
automatic switches/thermostats video/compact disc reading head camera auto-focus	active control of structures vibration suppression active materials surgical equipment robotics space structure deployment	atomic force microscope

in terms of force, displacement, stiffness, size, mass, response time (or operating frequency), power, efficiency and resolution. These must be matched to the performance characteristics of an actuator in order to determine whether the actuator can give the *performance* required for the application. Requirements such as cost, durability, maintenance, and environmental impact are less precisely defined and are not considered here. This article provides an overview of the range of actuation systems, gives a quantitative comparison of their performances, and presents examples of a systematic selection procedure for actuators. The mechanical performances of man-made actuators are compared with the corresponding performances of naturally occurring systems; this comparison is relevant to the design of prosthetic devices, where it is necessary to match the performance of a man-made system with that of a natural system.

(*a*) *Definitions*

Actuators offer a wide variety of performance and operate in many different ways. For the present analysis an actuator is defined to be a controllable work-producing machine. In order to provide a quantitative means of comparison, the scope of this article is restricted to actuators which operate in a linear fashion, causing a finite change in length. The study does not deal with motors which are able to produce (in principle) infinite displacements—these could be considered in a separate analysis. A number of definitions which assist the comparison are listed in table 2.

2. Performance characteristics of actuators

The maximum actuation stress, σ_{max}, and maximum actuation strain, ϵ_{max}, are basic characteristics of an actuator. For a given size of actuator they limit the force and displacement. Alternatively, given the design values for the required forces and displacements, the size and shape of a suitable actuator may be estimated. The stress versus strain (σ–ϵ) characteristic of an actuator is not a single curve; it is a family

Table 2. *Definitions*

performance characteristic	definition
actuation stress (σ)	The applied force per unit cross-sectional area of an actuator.
maximum actuation stress ($\sigma_{\max}$)	The maximum value of actuation stress in a single stroke which produces maximum work output.
actuation strain (ϵ)	The nominal strain produced by an actuator; an actuator of initial length L extends to a total length of $(1 + \epsilon)L$.
maximum actuation strain ($\epsilon_{\max}$)	The maximum value of actuation strain in a single stroke which produces maximum work output.
actuator density (ρ)	The ratio of mass to initial volume of an actuator. (We neglect the contribution to mass from power supplies, external fixtures and peripheral devices. For example, in the mass of a hydraulic cylinder, we include the working fluid and the cylinder, but neglect the compressor, servo-valve, cooling system and mounting fixtures.)
actuator modulus (E)	The ratio of a small increment in σ to the corresponding small increment in ϵ when the control signal to an actuator is held constant. (In general this differs from the measured modulus $\mathrm{d}\sigma/\mathrm{d}\epsilon$ which depends upon the control signal.)
volumetric power (p)	The mechanical power output per unit initial volume in sustainable cyclic operation.
efficiency (η)	The ratio of mechanical work output to energy input during a complete cycle in cyclic operation.
strain resolution ($\epsilon_{\min}$)	The smallest step increment of ϵ (order of magnitude approximations are given).

of curves which depend on the control signal and the external constraints. As an example, several stress versus strain curves available from a hydraulic actuator are shown in figure 1. If the work output per unit volume is maximized, then the σ–ϵ curves shown in figure 2 result.

The curves in figure 2 are based on a constant actuation stress for hydraulic and pneumatic actuators, and a linear relationship between actuation stress and strain in piezoelectric, magnetostrictive and thermal expansion actuators. For shape memory alloys, the σ–ϵ curve depends upon the material and the operating conditions. The shape shown in figure 2 corresponds to the general form found by a variety of experimenters (Bidaux *et al.* 1994; Shaw & Kyriakides 1995). The σ–ϵ curve for muscle is based on data in the literature of muscle physiology (Woledge 1985; McMahon 1984). The product $\sigma_{\max}\epsilon_{\max}$ is an estimate of the maximum work per unit volume in a single stroke. More precisely, a dimensionless stroke work coefficient C_s can be defined as the ratio of the maximum work done in a single stroke to the product $\sigma_{\max}\epsilon_{\max}$:

$$C_s = \int_0^1 \frac{\sigma}{\sigma_{\max}} \, \mathrm{d}\left(\frac{\epsilon}{\epsilon_{\max}}\right). \tag{2.1}$$
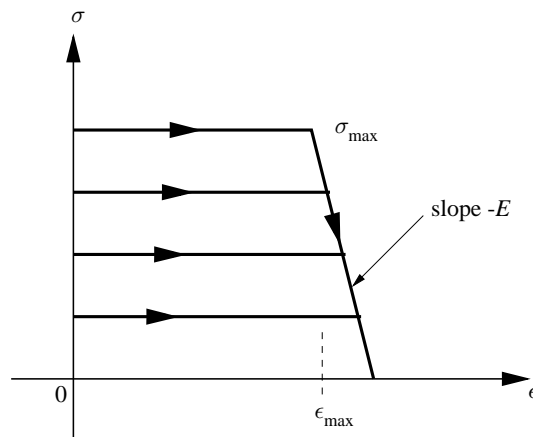
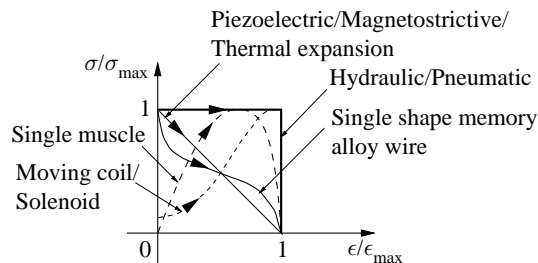Figure 1. Several stress versus strain characteristics for a hydraulic actuator.

Figure 2. Normalized single stroke stress versus strain curves. The curves show the
approximate shapes of the σ versus ϵ curves which maximize work per unit volume.

The coefficient C_s lies in the range zero to unity and is an efficiency measure of the
shape of the σ–ϵ curve.

Now consider *cyclic* operation of an actuator. Some actuators are intrinsically well
suited to cyclic operation and some are not. Shape memory alloys, solenoids, and
organic muscles typically provide a *single action* and require an external system to
reset them for cyclic operation. In shape memory alloys the resetting force provides
a severe limitation on cyclic operation. Approximate σ–ϵ curves for cyclic operation
producing maximum work per unit volume per cycle are shown in figure 3. For
simplicity, the curves for single acting systems are based on the action of a pair of
actuators operating antagonistically, as shown in figure 4.

The curves in figure 3 relate to low-frequency operation, where inertial and rate
limiting-effects are negligible. In this operating regime, the work available per cycle
is $4C_c\sigma_{\max}\epsilon_{\max}$, where C_c is a dimensionless cyclic work coefficient defined in cyclic
operation with maximum work output per cycle by:

$$C_c = \frac{1}{4} \oint \frac{\sigma}{\sigma_{\max}} \, \mathrm{d}\left(\frac{\epsilon}{\epsilon_{\max}}\right). \qquad (2.2)$$

The coefficient C_c lies in the range zero to unity and is an efficiency measure of
the shape of the σ–ϵ curve in cyclic operation. Values for C_s and C_c are given in
table 3.

At low frequency, the power per unit volume, p, is proportional to the frequency,
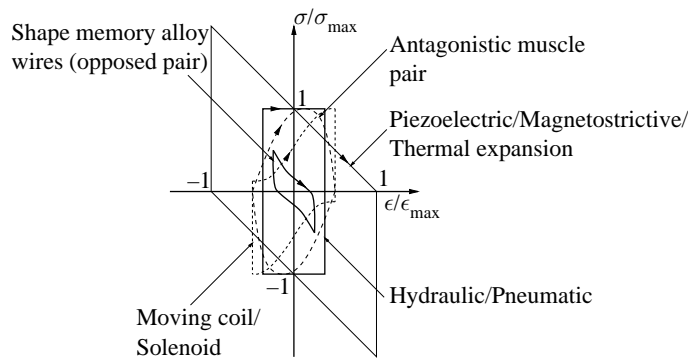f, of operation, giving $p = 4fC_c\sigma_{\max}\epsilon_{\max}$. At high frequency the performance ap-

Figure 3. Normalized cyclic stress versus strain curves. The figure shows the approximate shapes of the cyclic σ versus ϵ curves which maximize work per unit volume per cycle at low frequency.
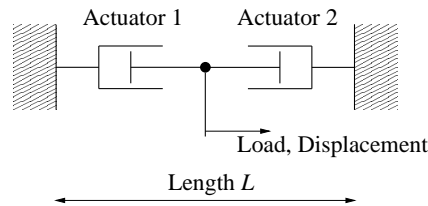

Figure 4. Antagonistic pair of actuators.

proaches a limit either in the form of a maximum power per unit volume, $p_{\max}$, or of a maximum frequency of operation, $f_{\max}$. The limit $f_{\max}$ is determined as follows. In devices where mechanical resonance places a limit on the operating frequency (piezoelectric and magnetostrictive devices), $f_{\max}$ is defined by the frequency of first resonance. This value depends on the size of the actuator, and the appropriate length dimension is based on the smallest length available using current manufacturing practice. In devices which rely on temperature change for actuation (shape memory alloy and thermal expansion) the value of $f_{\max}$ is based on the time required to transfer heat into and out of the actuator. For these devices it is found that heat transfer is limited by convection at the surface of the actuator, and the heat transfer coefficient is typically in the range 100–1000 W m^{-2} K^{-1}. The time required for heat transfer also depends on a length dimension and, as with resonance, the smallest available size has been used to give an approximate bound on frequency. In other devices frequency limits are taken from the sources referred to in the appendix. It is recognized that only an approximate bound on operating frequency can be given when discussing devices in such a general way. However, the resulting values for $f_{\max}$ vary by about seven orders of magnitude between different classes of actuator, and so an approximate analysis is sufficient to distinguish their performances. Above the frequency $f_{\max}$ some actuators are inoperable. Others suffer a reduction in power due to the inertia of moving parts; the amplitude of cyclic actuation drops to keep the inertial forces within the actuator's capability, and the power varies with frequency approximately as $1/f^2$.

The power limit $p_{\max}$ is based, in piezoelectric and magnetostrictive devices, on the maximum power to avoid overheating. In hydraulic and pneumatic actuators a limit is provided by the maximum sliding speed for the sealing system. The above analysis results in a power versus frequency characteristic, for a particular actuator, of the form shown in figure 5.
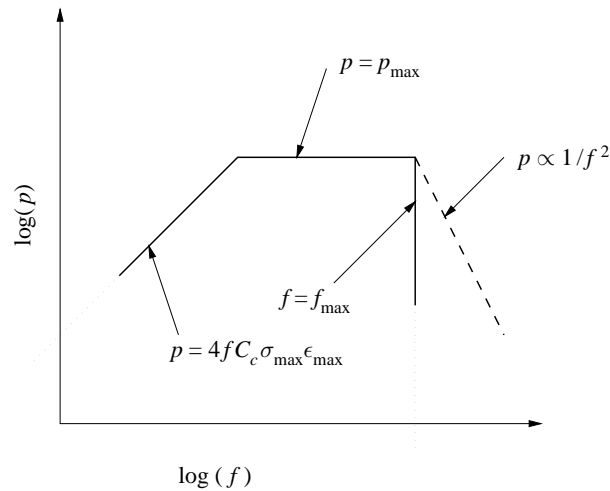
Figure 5. Typical shape of the volumetric power, p, versus frequency, f, limiting envelope for an actuator.

3. Actuator property charts

A systematic procedure for the selection of materials in engineering design, employing performance indices and material property charts, has been demonstrated by Ashby and co-workers (Ashby 1989, 1992; Ashby & Cebon 1993). When the characteristics of *actuators* are displayed on property charts, certain relationships between the different classes of actuators become evident. Consider, for example, a chart which displays the feasible combinations of actuation stress, σ, and actuation strain, ϵ, as shown in figure 6. The data in figure 6 are drawn from a database of actuator characteristics which is summarized in table 3. The values of σ and ϵ range over several decades, so the axes of the chart are logarithmic. Heavy lines show the locus of the values of maximum actuation stress versus actuation strain for each class of actuator. At low values of actuation strain, this locus follows the highest value of $\sigma_{\max}$ within the class. In some classes of actuator (shape memory alloys are an example) the highest values of $\epsilon_{\max}$ correspond to smaller values of $\sigma_{\max}$ and there is a boundary of approximately constant product $\sigma\epsilon$ which is finally cut off by the highest value of $\epsilon_{\max}$ in the class. Consequently, the heavy lines in figure 6 mark the upper right hand corner of the envelope of performance of each class of actuator. Actuators which give significant displacement per unit length lie towards the right of figure 6; they are naturally suited to applications where high stroke is required, as in the moving parts of plants, animals and machines. The actuators towards the top of figure 6 are suited to high force applications: hydraulic rams are used as presses in deformation processing, and shape memory alloy wires are used to press teeth into place and to seal vacuum pipe-work.

Presenting this information on logarithmic scales allows more to be shown. A straight line of slope -1 in figure 6 links points of constant $\sigma\epsilon$ product. Now, the stroke work available per unit volume has the form $C_s\sigma_{\max}\epsilon_{\max}$ and the values of C_s vary by less than a factor of four. Consequently, lines of slope -1 link classes of actuators with approximately the same volumetric stroke work. The sloping boundary of performance of classes of actuators such as shape memory alloys can be interpreted as a limitation on the available volumetric stroke work from that class. Shape memory alloy actuators operating at high values of actuation strain achieve a reduced actua-

Table 3. *Approximate ranges for the characteristics of mechanical actuators*

actuator type	maximum actuation strain $\epsilon_{\max}[-]$	maximum actuation stress $\sigma_{\max}$ (MPa)	modulus E (GPa)
low strain piezoelectric	5×10^{-6}–3×10^{-5}	1–3	90–300
high strain piezoelectric	5×10^{-5}–2×10^{-4}	4–9	50–80
piezoelectric polymer	2×10^{-4}–1×10^{-3}	0.5–5	2–10
thermal expansion (10 K)	9×10^{-5}–3×10^{-4}	20–50	70–300
thermal expansion (100 K)	9×10^{-4}–3×10^{-3}	200–500	70–300
magnetostrictor	6×10^{-4}–2×10^{-3}	90–200	40–200
shape memory alloy	7×10^{-3}–7×10^{-2}	100–700	30–90
moving coil transducer	1×10^{-2}–1×10^{-1}	4×10^{-3}–5×10^{-2}	4×10^{-5}–5×10^{-3}
solenoid	1×10^{-1}–4×10^{-1}	4×10^{-2}–1×10^{-1}	3×10^{-4}–1×10^{-3}
muscle	3×10^{-1}–7×10^{-1}	0.1–0.4	5×10^{-3}–2×10^{-2}
pneumatic	1×10^{-1}–1×10^{0}	0.5–0.9	5×10^{-4}–9×10^{-4}
hydraulic	1×10^{-1}–1×10^{0}	20–70	2–3

actuator type	maximum frequency $f_{\max}$ (s^{-1})	maximum power density $p_{\max}$ (W m^{-3})	density ρ (kg m^{-3})
low strain piezoelectric	5×10^{5}–3×10^{7}	1×10^{8}–1×10^{9}	2600–4700
high strain piezoelectric	5×10^{5}–2×10^{7}	9×10^{7}–5×10^{8}	7500–7800
piezoelectric polymer	1×10^{5}–1×10^{7}	$\approx 3 \times 10^{8}$	1750–1900
thermal expansion (10 K)	4×10^{-1}–9×10^{0}	$\approx 6 \times 10^{4}$	3900–7800
thermal expansion (100 K)	4×10^{-1}–9×10^{0}	$\approx 6 \times 10^{6}$	3900–7800
magnetostrictor	$\approx 3 \times 10^{7}$	1×10^{8}–7×10^{8}	6500–9100
shape memory alloy	2×10^{-2}–7×10^{0}	7×10^{5}–1×10^{8}	6400–6600
moving coil transducer	2×10^{4}–5×10^{4}	5×10^{5}–2×10^{6}	7000–7600
solenoid	5×10^{0}–8×10^{1}	1×10^{4}–4×10^{4}	3800–4400
muscle	5×10^{1}–5×10^{2}	$\approx 5 \times 10^{5}$	1000–1100
pneumatic	5×10^{1}–3×10^{2}	$\approx 5 \times 10^{6}$	180–250
hydraulic	5×10^{1}–3×10^{2}	$\approx 5 \times 10^{8}$	1600–2000

| actuator type	efficiency $\eta[-]$	resolution $\epsilon_{\min}[-]$	stroke work coefficient $C_{s}[-]$	cyclic power coefficient $C_{c}[-]$
low strain piezoelectric	> 0.9999	10^{-9}–10^{-8}	≈ 0.5	≈ 1
high strain piezoelectric	0.90–0.99	10^{-8}–10^{-7}	≈ 0.5	≈ 1
piezoelectric polymer	0.90–0.95	10^{-8}–10^{-7}	≈ 0.5	≈ 1
thermal expansion (10 K)	2×10^{-5}–3×10^{-4}	10^{-5}–10^{-4}	≈ 0.5	≈ 0.25
thermal expansion (100 K)	2×10^{-4}–3×10^{-3}	10^{-5}–10^{-4}	≈ 0.5	≈ 0.25
magnetostrictor	0.80–0.99	10^{-7}–10^{-6}	≈ 0.5	≈ 1
shape memory alloy	0.01–0.02	10^{-5}–10^{-4}	0.3–0.6	0.05–0.08
moving coil transducer	0.50–0.80	10^{-6}–10^{-5}	0.5–1.0	0.25–0.5
solenoid	0.50–0.80	10^{-4}–10^{-2}	0.5–1.0	0.25–0.5
muscle	0.20–0.25	10^{-4}–10^{-2}	0.5–0.7	0.25–0.35
pneumatic	0.30–0.40	10^{-5}–10^{-4}	≈ 1.0	0.4–0.5
hydraulic	0.90–0.98	10^{-5}–10^{-4}	≈ 1.0	0.4–0.5
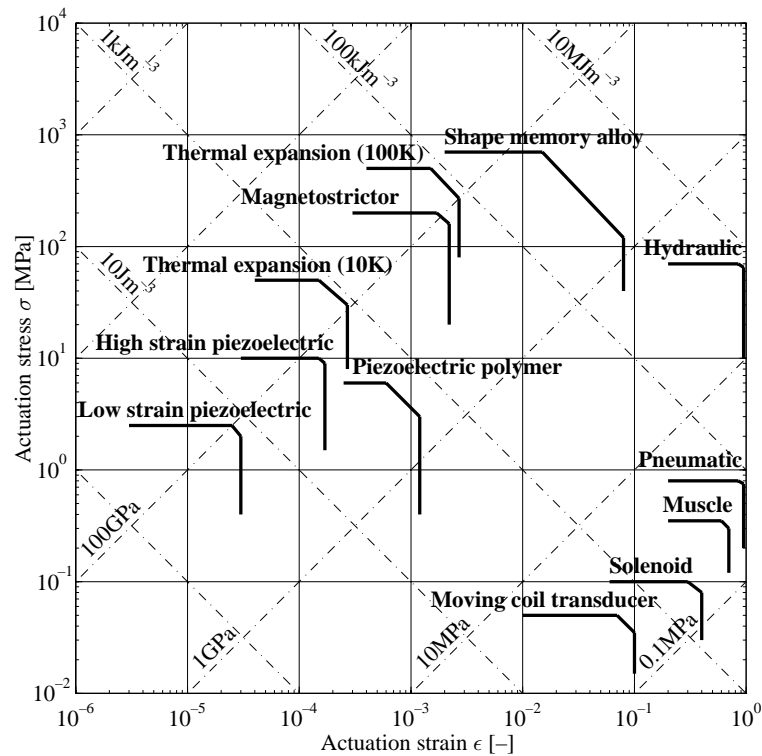
Figure 6. Actuation stress, σ, versus actuation strain, ϵ, for various actuators. Heavy lines bound the upper limits of performance.

tion stress because there is a constant quantity of energy per unit volume available from the martensitic transformation which drives the actuator.

Towards the top right of the diagram are actuators which are naturally suited to energy limited tasks, such as lifting weights, accelerating or propelling masses, or deforming stiff elastic structures. Figure 6 presents this information in a way which allows for quantitative comparisons. For example, there is currently interest in using piezoelectric devices to deform structural members such as plates and rods. There are severe limitations on the achievable work from a piezoelectric device: hydraulic or shape memory alloy systems can produce about four orders of magnitude greater work per unit volume than piezoelectric devices. The piezoelectric devices offer other advantages in that they can be distributed around a structure or embedded within it relatively easily.

Contours of constant stroke work in figure 6 also suggest potential competition between different systems. For example, solenoids, at about 1–5 kJ m^{-3} compete with thermal expansion actuators using a temperature change of between 10 K and 100 K. However, the stroke offered by thermal expansion is small, and a mechanism is required in order to develop strokes (and commensurate smaller forces) which match those of solenoids. In general an efficient mechanism can move the performance of an actuator along a line of constant $\sigma\epsilon$. The mechanical advantage required can be read directly from figure 6—it is of the order of 10^5. In practice, thermal actuators in the form of bimetallic strips (which amplify strain) are an alternative to solenoids in domestic appliances, when high frequencies are not required.

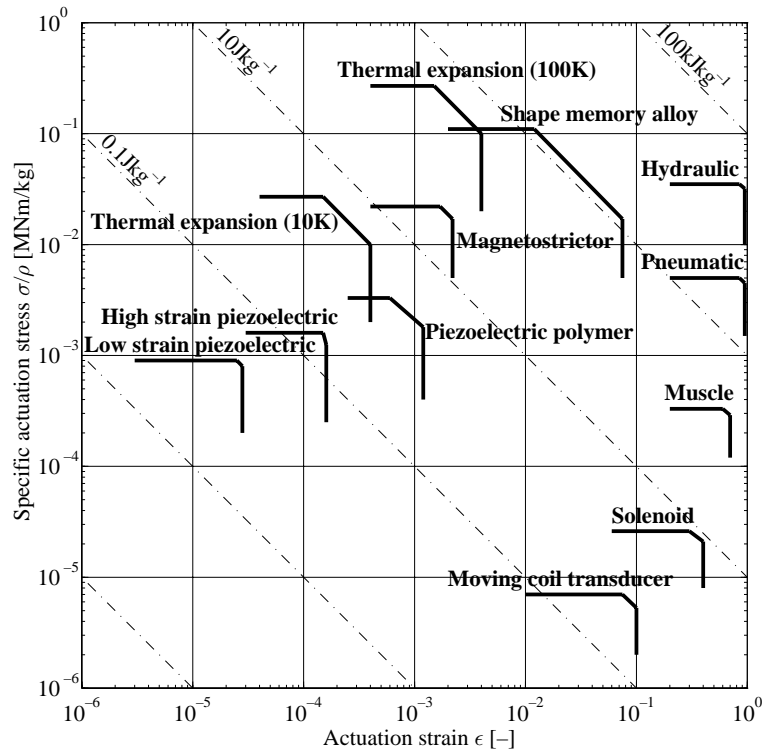Lines of slope +1 in figure 6 link actuators with the same value of σ/ϵ. This is a

Figure 7. Specific actuation stress, σ/ρ, versus actuation strain, ϵ, for various actuators. Heavy lines bound the upper limits of performance.

modulus-like quantity, although it is not always the same as the modulus E defined in § 1 *a*. Several classes of actuators are clustered around the line $\sigma_{\max}/\epsilon_{\max} = 100$ GPa. These are the metals and ceramics which generate force through elastic constraint. They have a high intrinsic modulus and are well-suited to open-loop control tasks. Actuators towards the lower right of figure 6 are more likely to require closed-loop control. Figure 6 shows that pneumatic actuators give a similar performance to muscle in single stroke operation.

The comparisons made so far have been made on the basis of actuators of equal size. Similar comparisons on the basis of equal mass are made possible by a chart of σ/ρ versus ϵ as shown in figure 7; it helps when choosing actuators for applications in which weight or inertial force is to be minimized. A note of caution: the density ρ, specified in figure 7, relates to the actuator itself, and not to peripheral devices such as power supplies.

A graphical presentation of the characteristics of actuators can also be used to assess performance in terms of *resolution*. This is significant for two reasons. First, the number of distinct positions through which an actuator must be able to step varies from application to application. An actuator for opening or closing a lock needs only two stable positions, whereas an actuator for controlling the position of a cutting tool may need several thousands. Second, the minimum step size required depends on the application. A compact disc reading head may need position control to within a wavelength of visible light, whereas a similar size of flow control valve may need a resolution of only tenths of a millimetre. These two aspects of actuators are displayed in figure 8. In the present study it is assumed that the actuator itself,
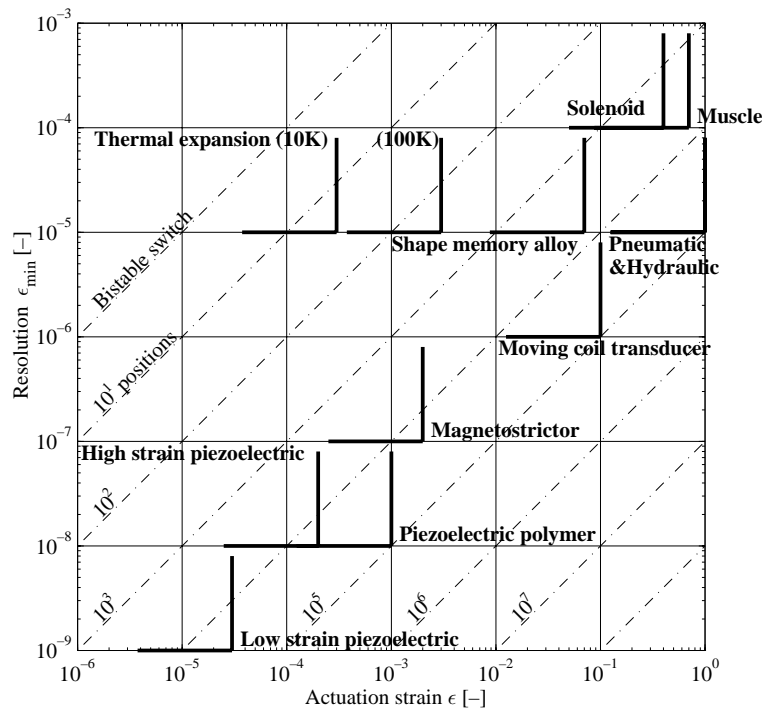
Figure 8. Strain resolution $\epsilon_{\min}$ versus actuation strain ϵ for various actuators. Heavy lines
bound the limits of performance.

rather than its control system, limits the strain resolution. The values of strain resolution are approximate; however, the enormous range of values allows meaningful conclusions to be drawn. Classes of actuators towards the right of the figure are suited to high stroke applications, and classes towards the bottom of the figure are suited to applications where control of small displacements is required. Lines of slope +1 link classes of actuators with the same numbers of distinct accessible positions within their strokes. Systems towards the upper left of the figure are suitable as simple switches, or where few discrete positions are required; those towards the lower right are suited to situations where continuous position control is desirable.

Where an actuator must operate cyclically, considerations of frequency, power and efficiency become relevant. Figure 9 allows for a comparison of frequency, power and bandwidth. It is evident that piezoelectric and magnetostrictive actuators are capable of producing high volumetric power when operated at sufficiently high frequency. At low frequency, hydraulic and shape memory alloy systems have the highest values of volumetric power, which is consistent with their high values of volumetric stroke work. Lines of slope +1 in figure 9 link actuators which can produce equal volumetric work output in each cycle. Figure 9 shows that pneumatic systems and muscle share similar power and bandwidth characteristics; figure 6 showed their similar performances in single stroke actuation. This suggests that pneumatic actuators are a suitable artificial substitute for muscle. Input power, output power and efficiency can be compared using figure 10. The low efficiency of thermally operated actuators such as thermal expansion devices and shape memory alloys is significant where energy usage is to be minimized. Lines of slope +1 in figure 10 link actuators which use equal input power per unit volume.
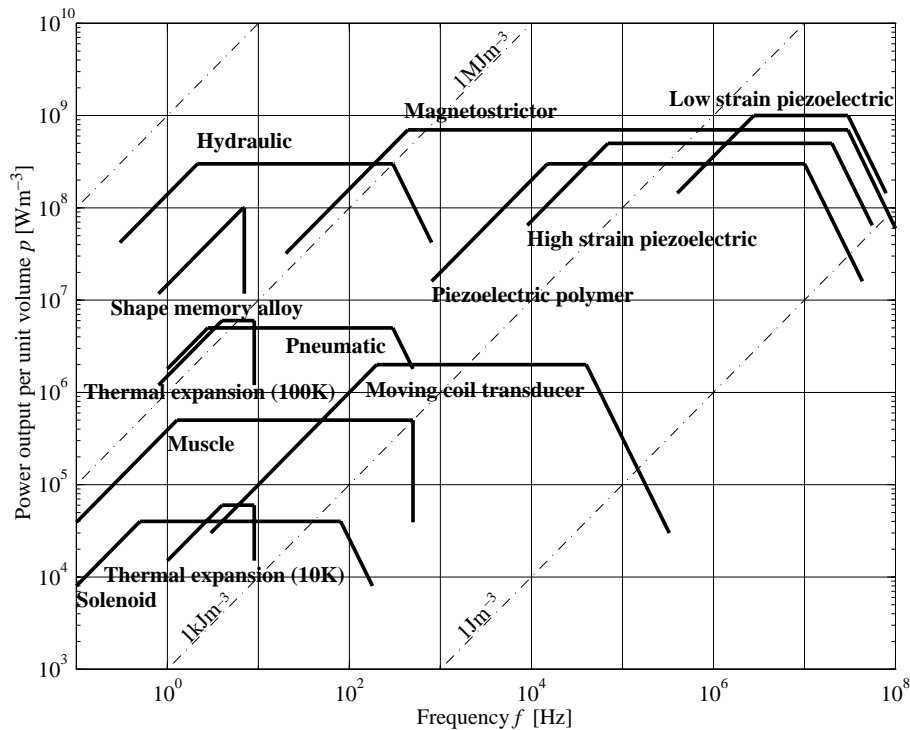
Figure 9. Volumetric power, p, versus frequency, f, for various actuators.

4. Performances indices for actuators

The performance index for an actuator is the combination of actuator character-istics which measures its effectiveness in performing a given function. Performance indices guide the selection of an appropriate type of actuator for a given application. The approximate nature of the performance characteristics suggests that a selection of this type should be used as a guideline only. In selecting a system based on the value of some performance index, systems which lie within about an order of magni-tude of each other in performance should be considered as potential competitors for the same application. Ingenuity in design can contribute significantly to the perfor-mance of a particular actuator. Performance indices are now derived for some generic actuation tasks.

(*a*) *Selection of a compact single stroke actuator*

Consider the following generic problem: an actuator is required to be capable of providing a prescribed force F, or/and a prescribed displacement δ, not necessarily simultaneously. The volume, V, of the actuator is to be minimized. The actuator may work through a simple mechanism such as a lever. The actuator has length L, cross-sectional area A and mechanical advantage r, as shown in figure 11.

There are constraints both on the length, L, of the actuator, and on its cross-sectional area, A. The constraint on length arises because the actuator must achieve displacement, δ, but has limited strain, ϵ, so $\delta \leqslant L\epsilon_{\max}r$. Similarly, the prescribed force, F, must be achieved using limited stress, σ, so $F \leqslant A\sigma_{\max}/r$. Volume, $V = AL$,
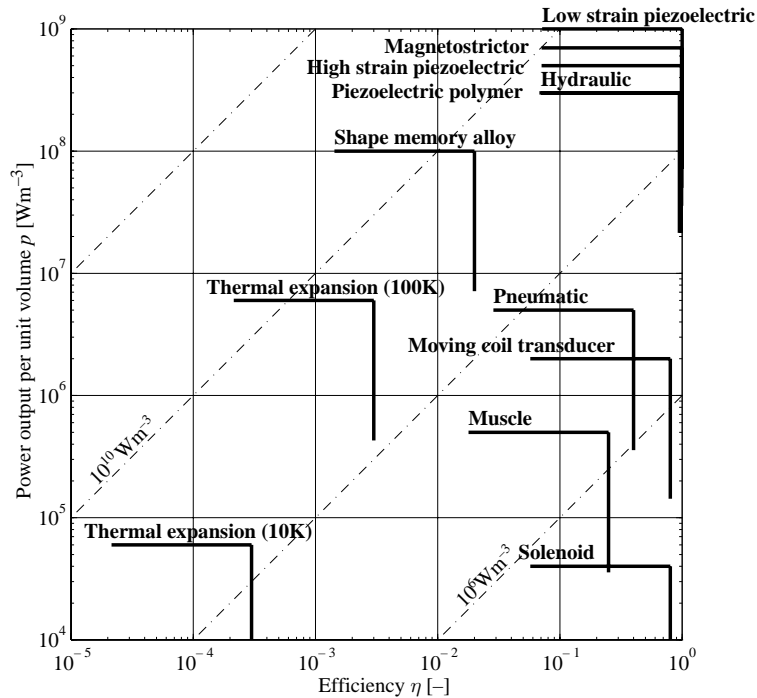
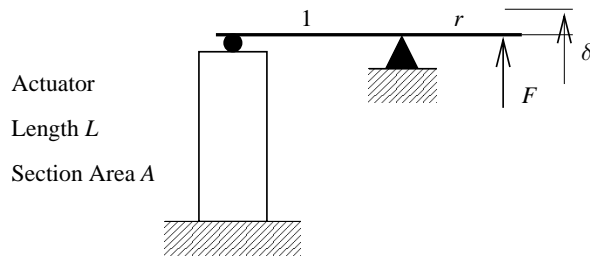Figure 10. Volumetric power, p, versus efficiency, η, for various actuators.

Figure 11. Single stroke actuator with a lever of mechanical advantage r.

is to be minimized. Substituting the two constraints gives

$$V \geqslant \frac{F\delta}{\sigma_{\max}\epsilon_{\max}}. \tag{4.1}$$

To minimize the volume, the product $\sigma_{\max}\epsilon_{\max}$ must be maximized; $\sigma_{\max}\epsilon_{\max}$ is the performance index for this problem. Figure 6 suggests that hydraulic systems or systems based on shape memory alloys would be selected where force and displacement are the only criteria. If the mass of the actuator were to be minimized, $\sigma_{\max}\epsilon_{\max}/\rho$ would become the performance index. Figure 7 shows that pneumatic systems also become competitors for this type of task. The use of a mechanism between an actuator and its load does not affect this selection. In applications such as lifting, propelling or accelerating a given mass and deforming a spring-like load, it is desirable to maximize the work available per unit volume in a single stroke. The available work per unit volume, $C_s\sigma_{\max}\epsilon_{\max}$, is then the performance index. Table 4 summarizes these results.

Table 4. *Summary of performance indices for single stroke actuators*

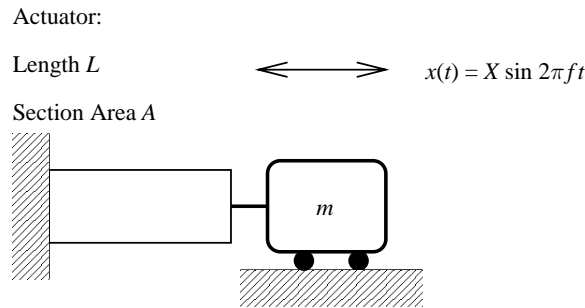| task	to be minimized	performance index
fixed force, fixed stroke	size	$\sigma_{\max}\epsilon_{\max}$
fixed energetic task	size	$C_s\sigma_{\max}\epsilon_{\max}$
fixed force, fixed stroke	weight	$\sigma_{\max}\epsilon_{\max}/\rho$
fixed energetic task	weight	$C_s\sigma_{\max}\epsilon_{\max}/\rho$

Actuator:

Length L

Section Area A

$x(t) = X \sin 2\pi f t$

Figure 12. Cyclic actuator oscillating a mass m at frequency f.

(*b*) *Selection of a compact cyclic actuator*

A typical cyclic operation is to provide a prescribed cyclic displacement at a given frequency. Consider, as a representative example, the problem of selecting an actuator to oscillate a mass, m, at a frequency, f, with an amplitude, X, as shown in figure 12. The volume, V, of the actuator is to be minimized.

In this example there are constraints on frequency, length and area. The frequency constraint is that $f_{\max}$ must be greater than the required operating frequency, f. The constraint on length arises because displacement, X, must be achieved with limited strain, ϵ, which gives $\epsilon_{\max}L \geqslant X$. The cross-sectional area is constrained because the actuator must provide sufficient force to accelerate the mass, m, using a limited actuation stress. At any point in the cycle the required force is $\sigma A = m\ddot{x}(t)$, where $x(t) = X \sin 2\pi f t$. The constraint is then $\sigma_{\max}A \geqslant 4\pi^2 f^2 X m$. Writing $V = AL$ and substituting for A and L using the constraints gives

$$V \geqslant \frac{4\pi^2 f^2 X^2 m}{\sigma_{\max}\epsilon_{\max}}. \tag{4.2}$$

As with single-stroke actuation, the best performance is achieved by selecting the actuator which has the maximum value of $\sigma_{\max}\epsilon_{\max}$, but now the selection is frequency dependent. Figure 13 indicates that at frequencies below about 10 Hz shape memory alloys and hydraulic systems would be competitors. Above 10 Hz but below about 300 Hz hydraulic systems would dominate. Above about 300 Hz magnetostrictors may be used, in principle, for an actuator of minimum volume. In practice, minimum volume may not be the key design requirement. For example, the cost per unit volume of magnetostrictive materials is currently far higher than that of electromagnetic actuators such as solenoids and moving-coil transducers. The result is that for a low cost shaker electromagnetic actuators are a practical alternative to magnetostrictors for frequencies up to about 50 kHz.
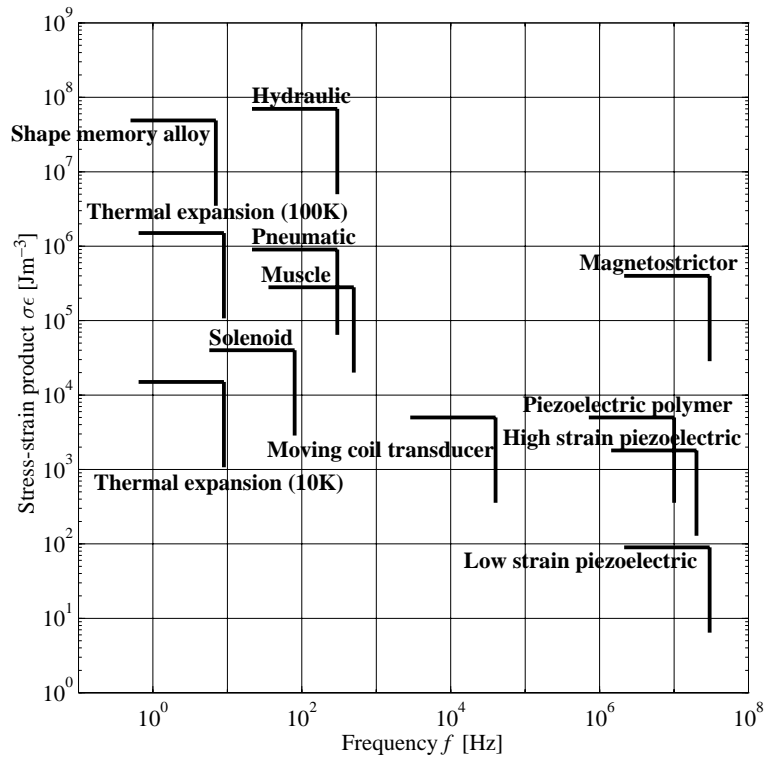
Figure 13. Stress–strain product $\sigma\epsilon$ versus frequency f for various actuators.

(c) *Selection of an actuator for vibration damping*

Currently, there is widespread research activity into the potential application of cyclic actuators for the control of vibration in structures. We consider, as a case study, the selection of an actuator of minimum volume in order to damp a free vibration. Consider a simplified system consisting of an undamped mechanical oscillator with stiffness, s, vibrating with initial amplitude x_0. An actuator of cross-sectional area A and length L is to be attached as shown schematically in figure 14. The actuator is intended to remove energy from the oscillating system, bringing it to rest in a prescribed number of cycles, N, where $N \gg 1$. What is the minimum volume actuator which can achieve this task?

The maximum stored elastic energy in the oscillator at amplitude x is $E = \frac{1}{2}sx^2$. During each cycle the amount of energy which the actuator can remove depends on its cyclic stress versus strain characteristic, its volume and the amplitude of oscillation. Stress versus strain characteristics for actuators operating at maximum cyclic strain amplitude are shown in figure 3. An example of the stress versus strain characteristic with varying strain amplitude is shown in figure 15.

In the nth cycle the oscillation has amplitude x and the actuator dissipates work of magnitude $AL \oint \sigma(\epsilon'; \epsilon)\, d\epsilon'$ where the strain amplitude of the current cycle $\epsilon \equiv x/L$, and $\sigma(\epsilon'; \epsilon)$ is the stress corresponding to a strain ϵ' in a cycle with strain amplitude ϵ. The dissipation per cycle is

$$\frac{d}{dn}(\tfrac{1}{2}sx^2) = -AL \oint \sigma(\epsilon'; \epsilon)\, d\epsilon', \tag{4.3}$$
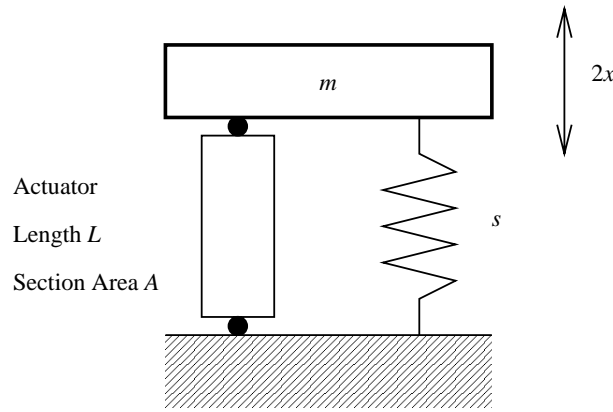
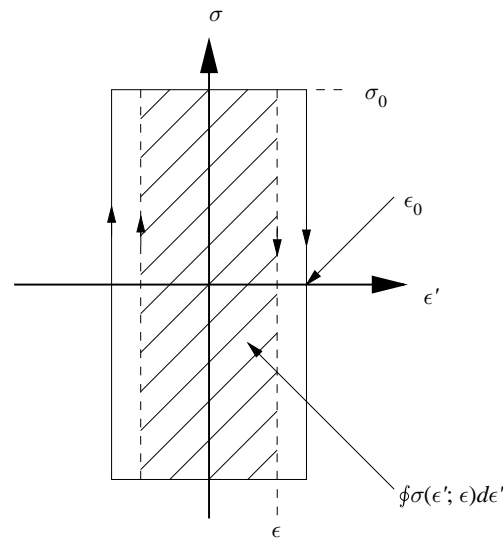Figure 14. An actuator used to damp a simple oscillator.

Figure 15. Cyclic stress versus strain characteristic for hydraulic and pneumatic actuators at varying strain amplitude.

which, on substituting for $x = \epsilon L$ and integrating, may be rearranged to the form

$$\int_0^N \frac{A}{sL}\,\mathrm{d}n = \frac{AN}{sL} = \int_0^{\epsilon_0} \left(\epsilon \bigg/ \oint \sigma(\epsilon';\epsilon)\,\mathrm{d}\epsilon' \right)\mathrm{d}\epsilon. \tag{4.4}$$

The right-hand side of equation 4.4 is a characteristic of the actuator and is a function of the initial strain amplitude, ϵ_0. It is convenient to express this in non-dimensional form by introducing a damping coefficient C_d defined as

$$\frac{1}{C_d(\epsilon_0,\sigma_0)} = 4\frac{\sigma_0}{\epsilon_0}\int_0^{\epsilon_0} \left(\epsilon \bigg/ \oint \sigma(\epsilon';\epsilon)\,\mathrm{d}\epsilon' \right)\mathrm{d}\epsilon, \tag{4.5}$$

where σ_0 is the initial stress amplitude. The damping coefficient is in the range zero to unity. Setting $\sigma_0 = \sigma_{\max}$ and ϵ_0 equal to the maximum cyclic strain amplitude which can be achieved, gives maximum values for C_d. For hydraulic and pneumatic actuators the characteristic is similar to the one shown in figure 15, for which C_d

equals unity. Piezoelectric, magnetostrictive and thermal expansion devices also have $C_d \approx 1$. For shape memory alloy wires arranged as an opposed pair C_d is approximately 0.2–0.3. The volume, V, of the actuator follows from equation 4.4:

$$V = AL = \frac{sx_0^2}{4NC_d\sigma_{max}\epsilon_0}.$$ (4.6)

Volume is minimized by selecting the actuator with the maximum value of the performance index $C_d\sigma_{max}\epsilon_0$.

In a practical example, such as damping vibration at the engine mounting of a large road vehicle, constraints limit the choice of actuator. Space constraints would limit the length L to about 0.1 m, so that a vibration of amplitude 1 mm would constrain the selection to actuators capable of cyclic strain amplitude ϵ_0 greater than 10^{-2}. This limits the selection to shape memory alloy, hydraulic, pneumatic and solenoid actuators. A constraint on frequency, $f_{max} \geqslant 100$ Hz, would eliminate shape memory alloys. Of the remaining choices, hydraulic actuators have by far the highest value of $C_d\sigma_{max}\epsilon_0$, resulting in the most compact design.

5. Conclusions

Performance characteristics which allow for comparison of a wide range of actuators have been developed. The penalty for such a broad comparison is a loss of precision—some aspects of the behaviour of individual actuators have had to be neglected, and it has been assumed that the properties of actuators are independent of frequency and scale. In practice, properties are frequency dependent, and scale effects, such as the dominance of frictional forces and surface energies in small devices, influence the choice of actuator. These aspects could be included in a more detailed analysis. The benefit of the simplifications which have been made is an overview which helps in selecting the most appropriate class of actuator for a given mechanical task. In the early stages of the design of an active system, all actuators should be considered; failure to do so may result in a missed opportunity. Graphical presentation of actuator characteristics helps to highlight where the opportunities lie for new devices or substitute devices with improved performance. Selection of an actuator on the basis of a performance index gives a systematic, if approximate, way of approaching design problems.

Performance characteristics have been estimated from manufacturers' data and from simple physical models of performance limitation. It must be recognized that such models contain assumptions about what is feasible both in design and in manufacture. It would not be surprising if some of the limits given proved to be unattainable in practical devices, or came to be surpassed through ingenious design.

To achieve the overview, actuators have been grouped into generic classes ('hydraulic', 'piezoelectric' and so on); the data which appear in table 3 and are plotted on the charts characterize whole classes and are necessarily approximate. Higher precision and more discriminating selection would be possible if the data for individual actuators within each class were stored and plotted. This method, particularly if implemented in software, could allow rapid identification of candidates for any given actuation need.

Appendix A. Actuation systems

In the following sections a brief description is given of each of the classes of actuators and of the limitations to their performances.

(*a*) *Piezoelectric actuators*

Piezoelectric materials strain when an external electric field e is applied. A variety of phenomena exist which include piezoelectricity ($\epsilon \propto e$), electrostriction ($\epsilon \propto e^2$) and ferroelectricity (ability to retain a remnant polarisation when $e = 0$). The phenomenon exhibited by a particular material depends on its structural symmetry and the temperature relative to its Curie temperature T_c. Above T_c the stable structure is neither ferroelectric nor piezoelectric. There is an extensive literature which provides an introduction to these materials (Xu 1991; Wang *et al.* 1987). For the present purposes, three groups of electrical materials are identified. *Low strain piezoelectrics* such as Quartz (SiO_2), Lithium Niobate ($LiNbO_3$) and Lithium Tantalate ($LiTaO_3$) are typically used as single crystals. They strain by up to about 3×10^{-5} upon application of an electric field. *High strain piezoelectrics* are dominated by lead zirconate titanate alloys ($PbZr_xTi_{1-x}O_3$, known as PZT) whose properties can be tailored by varying the alloy composition, and by introducing dopants. Piezoelectric actuation strains of up to about 2×10^{-4} are feasible. The efficiency of PZT ceramics is relatively low by comparison with the low strain piezoelectrics. *Piezoelectric polymers* such as polyvinylidene fluoride ((CH_2CF_2-)$_n$, known as PVDF) and related polymers can strain by up to about 1×10^{-3}; their properties can be modified by directional forming and they possess high formability. Their modulus (2–10 GPa) is about two orders of magnitude less than that of piezoelectric ceramics.

In all of the piezoelectric-type materials the maximum strain, $\epsilon_{\max}$, is limited by the tolerable level of electric field. An electric field of strength above the *coercive field* results in dramatic changes in properties such as the reversal of polarization in ferroelectric materials. In practice, field levels are often limited to a lower value to avoid mechanical and electrical fatigue. An actuation stress is generated by constraining the material from changing shape. In piezoelectric materials this can be modelled as linear elastic behaviour giving rise to a maximum actuation stress $\sigma_{\max} = E\epsilon_{\max}$ when the actuator is fully constrained. The maximum operating frequency is limited by mechanical resonance; $f_{\max} = c/2L$, where $c \approx \sqrt{E/\rho}$ is the mechanical wave speed and L is the length of the actuator. This limit is size dependent and so the absolute maximum operating frequency is limited by the ability to manufacture actuators of small size. A practical limit of 10^{-4} m is used here. The maximum power per unit volume $p_{\max}$ is limited by heat dissipation, and the need to maintain the device at a tolerable temperature (typically well below the Curie temperature to avoid rapid fatigue). The recommended power limits given by suppliers provide a useful guide. In the absence of this information, an approximate power limit can be derived based on the efficiency, and using air convection to remove heat from the device. Performance characteristics have been compiled from the literature of material suppliers (Morgan Matroc Ltd, Piezo Kinetics Inc., Channel Industries Inc., Piezo Systems Inc., Valpey-Fisher Corporation), and from other sources (Ikeda 1990; Wang *et al.* 1987; Moulson & Herbert 1990).

(*b*) *Shape memory alloy actuators*

The mechanism of actuation in shape memory alloys is a temperature-induced phase change which produces a significant shear strain on heating above the trans-

formation temperature. This effect has given rise to a variety of applications (Duerig 1990). High values of $\sigma_{\max}$ (up to about 7×10^8 N m^{-2}) and $\epsilon_{\max}$ (up to about 7×10^{-2}) can be achieved in nickel–titanium alloys of approximately equiatomic composition. Other shape memory alloys exist. In the copper–aluminium alloy system several alloys exhibit shape memory but the performance is relatively poor by comparison with Ni–Ti alloys. Iron–manganese–silicon alloys provide a potential low cost competitor, best suited to single-stroke operation because of their high hysteresis. For actuation, shape memory alloys are often used in the form of a wire or foil which reduces in length when heated, and can be returned to its original length by cooling and then stretching. Heating can be achieved by electrical resistance in shape memory alloy wires, with the resulting tensile forces providing a *single acting* actuator. For cyclic operation, a mechanism is required to reset the actuator. This imposes limitations on $\sigma_{\max}$ and $\epsilon_{\max}$. In the present analysis, the resetting mechanism is assumed to be a second shape memory alloy actuator—alternative resetting mechanisms such as springs or weights are also common. In cyclic applications, the frequency of operation and the power output are limited by heat transfer, and the temperature change required for actuation. A temperature change of 15–30 K is usually required to activate shape memory alloys. However, to achieve a high actuation stress, a larger temperature change (about 100 K) is preferable. Electrical resistance heating can be achieved rapidly; cooling is the main limitation on operating frequency. The maximum frequency is dependent on the relevant length scale for cooling, which can be as small as about 0.5×10^{-4} m, based on thinnest readily available shape memory alloy wires. Using this value for the radius of a shape memory alloy wire and cooling by convection with surface heat transfer coefficient in the range 100–1000 W m^{-2} K^{-1} gives rise to a maximum operating frequency in the range 0.02–7 Hz. Performance data for shape memory alloys have been compiled from the literature of material suppliers (Advanced Materials and Technologies n.v., Thomas Bolton Ltd, Dynalloy Inc.), and from other sources (Smithells 1992; Duerig 1990).

(c) *Magnetostrictive actuators*

Magnetostrictive materials produce an actuation strain under the influence of an applied magnetic field, caused by the reorientation of magnetic domains. Although many materials display this phenomenon, the effect is generally small except in iron-lanthanide compounds. The most commonly used magnetostrictor is 'Terfenol' ($Tb_{0.3}Dy_{0.7}Fe_{1.9}$). The actuation strain in magnetostrictors is limited by magnetic saturation, and this provides a suitable upper limit $\epsilon_{\max}$. The other aspects of magnetostrictors' performance are analogous to those of piezoelectrics and the same method has been used to determine the maximum operating frequency and the maximum power per unit volume. Data for magnetostrictors are taken from McCurrie (1994) and Wohlfarth (1980).

(d) *Thermal expansion actuators*

Thermal expansion gives rise to a shape change which can be used for actuation, and is convenient when the control signal is available in the form of a temperature, such as in thermostatic applications. The actuation strain equals $\alpha \Delta T$, where α is the coefficient of thermal expansion and ΔT is the temperature change. The maximum value of the actuation strain, $\epsilon_{\max}$ is limited by the maximum allowable temperature change which depends upon the specific application. Temperature changes in the range 10–100 K are typical. The choice of material for an actuator based on thermal

expansion depends on the requirements of the actuator. When a large actuation strain is desired for a given temperature change, α is to be maximized and polymers perform well. Note that the actuation stress is achieved by constraining the material from changing shape. Thus a high value of $\sigma_{\max}$ is achieved by selecting a material which maximizes $E\alpha$, provided that $\sigma_{\max} = E\alpha\Delta T$ does not exceed the failure stress. If a high volumetric stroke work is required the selection criterion becomes $E\alpha^2$. Engineering alloys perform well in these last two cases. Operating frequencies are limited by the rate of heat transfer into and out of the device. Estimates for the frequency and power limitations for thermal expansion actuators can be carried out in the same manner as described above for shape memory alloys. The critical dimension for cooling is taken to be 10^{-4} m, consistent with the minimum thicknesses of readily available bimetallic strips (Kanthal AB).

(*e*) *Hydraulic and pneumatic actuators*

Hydraulic and pneumatic actuators provide force and displacement via the flow of a pressurized fluid. The actuation strain is limited only by the design of the actuator. A reasonable upper limit is $\epsilon_{\max} \approx 1$. The maximum actuation stress, $\sigma_{\max}$, is limited by the pressure of the working fluid. In hydraulics the difficulty of high-pressure containment begins to outweigh high pressure advantages at about 40–45 MPa (Brunell 1979) but higher pressures are feasible; proprietary systems have actuation stresses up to about 70 MPa (Enerpac Ltd). In pneumatics, the compressibility of the working fluid gives rise to both safety and energy efficiency limitations at pressures in excess of about 1 MPa. The main limitation on volumetric power is set by the maximum sliding speeds $v_{\max}$ which can be tolerated by the seals, as follows. The actuator can do work $4C_c\sigma_{\max}\epsilon_{\max}$ per unit volume in a single cycle during cyclic operation. The maximum power per unit volume is then $p_{\max} = 4C_c\sigma_{\max}\epsilon_{\max}v_{\max}/L$ where L is the initial length of the actuator. A low value of L gives rise to a high value of $p_{\max}$ and so reducing the length of a hydraulic or pneumatic actuator raises $p_{\max}$. Lengths on a scale smaller than about 10^{-1} m become impractical and $v_{\max}$ is limited to about $0.5\,\mathrm{m\,s^{-1}}$ (Brunell 1979). This limits $p_{\max}$ to $3 \times 10^8\,\mathrm{W\,m^{-3}}$ for hydraulics and to $5 \times 10^6\,\mathrm{W\,m^{-3}}$ for pneumatics.

(*f*) *Electromagnetic actuators*

Three forms of electromagnetic machines are commonly used as actuators: solenoids, moving coil transducers and motors. Solenoids consist of an electromagnetic coil and a high permeability rod; when energized the solenoid pulls on the rod via its magnetic field. Commercially available solenoids have a maximum actuation stress $\sigma_{\max}$ of about $1 \times 10^5\,\mathrm{N\,m^{-2}}$ and actuation strains of up to about 0.4. Performance data for the most common types of solenoid can be found in the data sheets of component suppliers. In cyclic operation it is necessary to provide a resetting mechanism, which, in the present analysis, is taken to be a second solenoid. Resetting mechanisms such as springs and weights could also be used. Solenoids are usually limited to low frequencies (less than 100 Hz) because of the inertia of the moving armature. Frequencies of up to 50 kHz are achievable in moving coil transducers since the moving part is a low-inertia current-carrying coil. Details of a wide variety of designs for moving coil transducers are available in the literature (Wavre & Thouvenin 1995; Borwick 1988; Greenwood 1965). Motors, which can be arranged to give (in principle) infinite displacements, are not considered here; an introduction to linear motors can be found in the references (Nasar & Boldea 1976).

(*g*) *Natural actuators*

The means of actuation employed by living organisms for locomotion and shape change provide a useful comparison with artificial machines. Actuation in animals is carried out predominantly using muscle. Muscle strains by thick fibres of myosin climbing along thin fibres of actin. The mechanism relies upon the ability of the cross bridges at the heads of the myosin molecules to change shape, detach and re-attach further along the actin fibres. The chemical energy which drives this action is released from adenosine triphosphate (ATP). Performance characteristics have been estimated on the basis of data in the literature of muscle physiology (Woledge 1985; McMahon 1984). Typically, muscle can achieve nominal strains of up to about 0.75 and stresses of about 0.3 MPa.

The source of actuation which allows plants to achieve rapid active shape changes (such as stamens which bend over to daub visiting insects with pollen) is different. This usually relies on stored elastic energy which is built up gradually, in advance of action (Simons 1992). Plants can control the concentration of positively charged ions such as K^+ in their cell sap by using ion pumps which transport ions across cell membranes. Osmosis drives water up the K^+ concentration gradient, resulting in a turgor pressure within plant cells of about 1 MPa and a volumetric strain of up to 0.50 (Wilkins 1984). Turgor pressure can be made to act against an elastic structural member which deforms and then returns rapidly to its original shape when the pressure is released. The resulting single stroke actuator can operate rapidly, but even the most rapid plant gyration requires a reset time of 90 s (Hart 1990). Consequently the power in cyclic operation is low.

(*h*) *Actuators based on material state changes*

Various actuation systems based on expansion or contraction during phase changes in materials are possible. The example of shape memory alloys has already been discussed. A further example is the wax actuator, which is based on the expansion of wax during melting, and is used in thermostatic devices. Wax-based actuators can achieve strains of up to about 40% and actuation stresses of about 1 MPa (Duerig 1990). The liquid–solid phase change in water gives rise to actuation which is known for its damaging effect in structures, termed 'frost-heave'.

References

Advanced Materials and Technologies n.v. *Properties of shape memory alloys.* Industrieweg 43 B-3540 Herk-de-Stad, Belgium.

Ashby, M. F. 1989 On the engineering properties of materials. *Acta Metall.* **37**, 1273–1293.

Ashby, M. F. 1992 *Materials selection in mechanical design.* Oxford: Pergamon.

Ashby, M. F. & Cebon, D. 1993 Materials selection in mechanical design. *J. Physique* **3**, 1–9.

Barrett, R. 1992 Active plate and wing research using EDAP elements. *Smart Mater. Struct.* **1**, 214–226.

Bidaux, J.-E., Yu, W. J., Gotthardt, R. & Månson, J.-A. 1994 Modelling of the martensitic transformation in shape memory alloy composites. In *Proc. 3rd Eur. Symp. on Martensitic Transformations, Barcelona, 14–16/9/1994.*

Borwick, J. 1988 *Loudspeaker and headphone handbook.* London: Butterworth.

Bothwell, C. M., Chandra, R. & Chopra, I. 1995 Torsional actuation with extension torsion composite coupling and a magnetostrictive actuator. *AIAA Jl* **33**, 723–729.

Brunell, R. 1979 *Hydraulic and pneumatic cylinders.* Trade & Technical.

Channel Industries, Inc. *Piezoelectric ceramic.* 839 Ward Drive, Santa Barbara, CA 93111, USA.

Crawley, E. F. & de Luis, J. 1987 Use of piezoelectric actuators as elements of intelligent structures. *AIAA Jl* **25**, 1373–1385.

Duerig, T. W. 1990 *Engineering aspects of shape memory alloys.* London: Butterworth–Heinemann.

Dynalloy, Inc. *Flexinol Actuator Wires.* 18662 MacArthur Blvd., Suite103, Irvine, CA 92715, USA.

Enerpac Ltd. *High force hydraulic tools* 13000 W. Silver Spring Drive, Butler, Wisconsin 53007, USA.

Furnya, Y. & Shimada, H. 1991 Shape memory actuators for robotic applications. *Mater. Design* **12**, 21–28.

Greenwood, D. C. 1965 Manual of electromechanical devices. New York: McGraw-Hill.

Hart, J. W. 1990 *Plant tropisms and other growth movements.* London: Unwin Hyman.

Ikeda, T. 1990 *Fundamentals of piezoelectricity.* Oxford University Press.

Kanthal AB *Thermostatic Bimetal.* Strip and Bimetal, Box 518, S-734 27 Hallstahammar, Sweden.

McCurrie, R. A. 1994 *Ferromagnetic materials: structure and properties.* London: Academic.

McDonald Schetky, L. 1991 Shape memory alloy applications in space systems. *Mater. Design* **12**, 29–32.

McMahon, T. A. 1984 *Muscles, reflexes and locomotion.* Princeton University Press.

Morgan Matroc Ltd *Piezoelectric ceramics.* Vauxhall Industrial Estate, Ruabon, Wrexham, Clwyd LL14 6HY, UK.

Moulson, A. J. & Herbert, J. M. 1990 *Electroceramics: materials, properties, applications.* London: Chapman and Hall.

Nasar, S. A. & Boldea, I. 1976 *Linear motion electric machines.* New York: Wiley.

Piezo Kinetics, Inc. *Piezoceramics application notes.* P.O. Box 756, Pine Street and Mill Road, Bellefonte, PA 16823, USA.

Piezo Systems, Inc. *Product Catalog.* 186 Massachusetts Ave., Cambridge, MA 02139, USA.

Shaw, J. A. & Kyriakides, S. 1995 Thermomechanical aspects of NiTi. *J. Mech. Phys. Solids* **43**, 1243–1281.

Shen, M.-H. H. 1994 Analysis of beams containing piezoelectric sensors and actuators. *Smart Mater. Struct.* **3**, 439–447.

Simons, P. 1992 *The action plant: movement and nervous behaviour in plants.* Oxford: Blackwell.

Smithells, C. J. 1992 *Smithells metals reference book*, 7th edn. London: Butterworth-Heinemann.

Thomas Bolton Ltd *Data sheet MJB/SMA/601/88.* P.O. Box 1, Froghall, Stoke-on-Trent, Staffs. ST10 2HF, UK.

Valpey–Fisher Corporation *Piezoelectric transducer crystals.* 75 South Street Hopkinton, MA 01748, USA.

Venison, G. S. 1986 A practical look at shape memory alloys' potential as a thermal actuator. *Mater. Design* **7**, 143–146.

Wang, T. T., Herbert, J. M. & Glass, A. M. 1987 *The Applications of ferroelectric polymers.* Blackie.

Wavre, N. & Thouvenin, X. 1995 Voice-coil actuators in space. In *Proc 6th European Space Mechanisms & Tribology Symp.*

Wilkins, M. B. 1984 *Advanced plant physiology.* London: Pitman.

Wohlfarth, E. P. 1980 *Ferromagnetic materials: a handbook on the properties of magnetically ordered substances.* Oxford: North-Holland.

Woledge, R. C. 1985 *Energetic aspects of muscle contraction*, ch. 1, 2. London: Academic.

Xu, Y. 1991 *Ferroelectric materials and their applications.* Amsterdam: North Holland.